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ADVANCES IN THE FIELD-THEORETIC UNDERSTANDING OF PION PRODUCTION IN NUCLEON-NUCLEON COLLISIONS

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We study the production amplitude for the reaction $NN \to NN\pi$ up to next-to-leading order in chiral perturbation theory. We show that the irreducible chiral loops at this order exactly cancel those terms that arise from the off-shell parts of the πN rescattering vertex. This cancellation is required for formal consistency of the whole scheme. The net effect of the inclusion of all next-to-leading order loops is to enhance the leading rescattering amplitude by a factor of 4/3 compared to phenomenological studies, bringing its contribution to the cross section for $pp \to d\pi^+$ close to the experimental value.

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Understanding the dynamics of pion production in nucleon-nucleon collisions near threshold is of significant importance. It is the first hadronic inelasticity of NN scattering at intermediate energies, thus we can only understand NN scattering if we understand $NN \to NN\pi$. It is also a necessary step to an understanding of isospin violation in few-nucleon processes[1,2], which provides a test for chiral perturbation theory (ChPT), and it is needed for a calculation of absorptive corrections to πd scattering[3]. When accurate data for the total cross-section close to threshold appeared in 1990[4], existing models[5,6] failed to describe the data by a factor of five to ten for the channel $pp \to pp\pi^0$ and a factor two for the channels $pp \to pn\pi^+$

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and $pp \to d\pi^+$. To cure this discrepancy, many mechanisms were proposed – for a recent review see Ref. 7.

In the reaction $NN \to NN\pi$ the momentum transfer $|\vec{p}|$ is large compared to the pion mass already at threshold: $|\vec{p}_{\rm thr}| = \sqrt{m_\pi M}$ with m_π (M) — the pion (nucleon) mass. This new scale has to be accounted for, which leads to modifications of Weinberg's counting scheme[9,10]. The expansion parameter in this case is[11,12,13,14]

$$\chi = \frac{|\vec{p}_{\rm thr}|}{M} = \sqrt{\frac{m_\pi}{M}}. \tag{1}$$

In Ref. 14 the large momentum scale was shown to promote some loops to lower order compared to Weinberg's original counting. The leading loops are shown in Fig.1; the loops (b)–(d) enter at next-to-leading order (NLO). The diagrams (a) in Fig.1 are reducible according to the common rules – the one-pion-exchange is regarded as part of the wave function. Therefore they were not included into the transition operator. The findings of Ref. 14 were:

- For the channel $pp \to pp\pi^0$ the sum of diagrams (b)–(d) of Fig. 1 canceled:

$$A^{1b+1c+1d}_{pp\to pp\pi^0} = \frac{g_A^3}{256 f_\pi^5}\,(-2+3-1)\;|\vec{p}| = 0; \tag{2}$$

- For the channel $pp \to d\pi^+$ the same sum gave a finite answer:

$$A^{1b+1c+1d}_{pp\to d\pi^+} = \frac{g_A^3}{256 f_\pi^5}\,(-2+3+0)\;|\vec{p}| = \frac{g_A^3|\vec{p}|}{256 f_\pi^5}. \tag{3}$$

The latter amplitude grows linearly with increasing final NN–relative momentum, which leads to a large sensitivity to the final NN wave function, once the convolution of those with the transition operators is evaluated. However, such a sensitivity is not allowed in a consistent field theory. This problem was stated in Ref. 15; to cure this, a new counterterm at leading order was proposed. However, such a structure would violate chiral symmetry. We show here how to resolve this problem. At the same time we shed new light on the concept of reducibility in pion reactions on few-nucleon systems. For the details see Ref. 8. Our central finding is that the diagrams (a) in Fig.1 in fact contain a genuine irreducible piece due to the energy dependence of the leading order (LO) $\bar{N}N\pi\pi$ vertex. This irreducible part should be considered along with the diagrams (b)–(d) of Fig. 1. Without loss of generality we work in

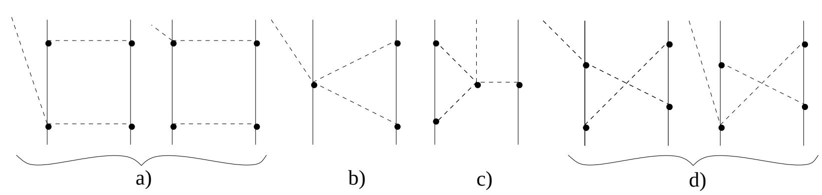

Fig. 1. Leading loop diagrams for $NN \to NN\pi$. Solid lines are nucleons, dashed lines are pions.

threshold kinematics, *i.e.*, we assume all final particles at rest and the initial relative momentum to be $\vec{p}$. We get for the full expression for the first diagram (a) of Fig.1 up to higher orders:

$$A^{1a1}_{pp\to d\pi^+} = i\frac{3g_A^3}{32f_\pi^5}\int\frac{d^4l}{(2\pi)^4}\frac{[l_0+m_\pi-(2\vec{p}+\vec{l})\cdot\vec{l}/(2M)]}{(l_0-\frac{m_\pi}{2}-\frac{(\vec{l}+\vec{p})^2}{2M}+i\epsilon)(-l_0+\frac{m_\pi}{2}-\frac{(\vec{l}+\vec{p})^2}{2M}+i\epsilon)} \times\frac{(\vec{l}\cdot(\vec{l}+\vec{p}))}{(l^2-m_\pi^2)((l+p)^2-m_\pi^2)}\ . \tag{4}$$

Let us rewrite the expression for the $\bar{N}N\pi\pi$ vertex in the numerator in Eq. 4 (note: since $\vec{p}^{\,2}/M$, $\vec{l}^{\,2}/M \sim m_\pi$, the recoil term contributes to the vertex at LO as well):

$$\left[l_0+m_\pi-\frac{(2\vec{p}+\vec{l})\cdot\vec{l}}{2M}\right]=\left[\left(l_0-\frac{m_\pi}{2}-\frac{(\vec{p}+\vec{l})^2}{2M}\right)+2m_\pi\right], \tag{5}$$

where we used threshold kinematics. The first term in the r.h.s. of Eq. 5 cancels the first nucleon propagator in Eq. 4. The corresponding piece of the diagram is irreducible and enters at NLO. We get for its contribution up to higher orders:

$$A^{1a1(\mathrm{irr})}_{pp\to d\pi^+}=-\frac{3}{4}\frac{g_A^3|\vec{p}|}{256f_\pi^5}\ , \tag{6}$$

where the label (irr) indicates that this is only the irreducible piece of the diagram. Analogous considerations apply to the second diagram of diagrams (a) of Fig. 1. Their contribution to the reaction $pp\to pp\pi^0$ is zero due to the isovector character of the leading $\bar{N}N\pi\pi$ vertex. Thus, one gets for the sum of all the NLO contributions:

$$\begin{aligned} A^{1a1(\mathrm{irr})+1a2(\mathrm{irr})+1b+1c+1d}_{pp\to d\pi^+} &= \frac{g_A^3}{256f_\pi^5}\left(-\frac{3}{4}-\frac{1}{4}-2+3+0\right)|\vec{p}|=0\ ,\\ A^{1a1(\mathrm{irr})+1a2(\mathrm{irr})+1b+1c+1d}_{pp\to pp\pi^0} &= \frac{g_A^3}{256f_\pi^5}\left(\ \ 0\ +0\ -2+3-1\right)|\vec{p}|=0\ . \end{aligned} \tag{7}$$

Thus, in both channels that contribute at the production threshold the sum of all irreducible NLO loops cancels. And no counterterm is necessary at this order, at variance with the claims of Ref. 15.

The remaining pieces in the expressions for $A^{1a}_{pp\to d\pi^+}$ exactly agree to the convolution of the LO rescattering contribution with the NN wave function, however, with the $\bar{N}N\pi\pi$ vertex put on-shell, *i.e.* instead of the commonly used[5] $3/2\ m_\pi$ in the vertex we have to use the value $2\,m_\pi$ – cf. Eq. 5. This enhances the dominating isovector πN-rescattering amplitude by a factor of 4/3, which leads to a good description of the experimental data for $pp\to d\pi^+$ — see Fig. 2 for the comparison.

To summarize, some pion production diagrams that seem reducible contain in fact irreducible pieces as a result of the energy dependence of the LO $\bar{N}N\pi\pi$ vertex. For the reaction $pp\to d\pi^+$ the net effect of the inclusion of all NLO loops is to enhance the LO rescattering amplitude by a factor of 4/3, bringing its contribution to the cross section close to the experimental value. The NLO contributions

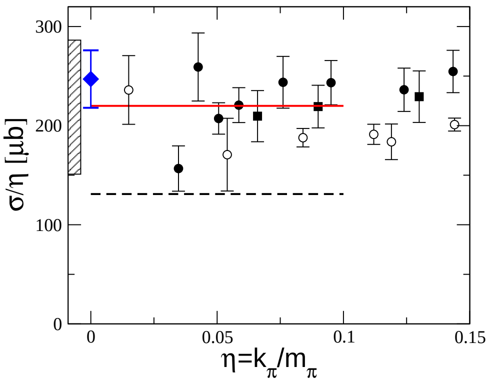

Fig. 2. Comparison of our results to experimental data for $pp \to d\pi^+$. The dashed curve shows the LO results. The solid line shows the results at NLO. The hatched bar shows the theoretical uncertainty for the NLO result. Data are from Refs. 16 (open circles), 17 (filled circles) and 18 (filled squares). The diamond shows result obtained from the width of pionic deuterium[19].

that would lead to a large sensitivity to the off-shell parameters cancel, which is a necessity for the formalism to be consistent.

References

1. U. van Kolck, J. A. Niskanen, and G. A. Miller, *Phys. Lett. B* **493**, 65 (2000).
2. A. Gårdestig *et al.*, *Phys. Rev. C* **69**, 044606 (2004).
3. V. Lensky, V. Baru, J. Haidenbauer, C. Hanhart, A. E. Kudryavtsev, and U.-G. Meißner, [arXiv:nucl-th/0608042].
4. H. O. Meyer *et al.*, *Phys. Rev. Lett.* **65**, 2846 (1990).
5. D. Koltun and A. Reitan, *Phys. Rev.* **141**, 1413 (1966).
6. G. A. Miller and P. Sauer, *Phys. Rev. C* **44**, 1725 (1991).
7. C. Hanhart, *Phys. Rep.* **397**, 155 (2004) [arXiv:hep-ph/0311341].
8. V. Lensky, V. Baru, J. Haidenbauer, C. Hanhart, A. E. Kudryavtsev, and U.-G. Meißner, *Eur. Phys. J A* **27**, 37 (2006) [arXiv:nucl-th/0511054].
9. S. Weinberg, *Phys. Lett. B* **251**, 288 (1990); *Nucl. Phys. B* **363**, 3 (1991).
10. S. Weinberg, *Phys. Lett. B* **295**, 114 (1992).
11. T. D. Cohen, J. L. Friar, G. A. Miller and U. van Kolck, *Phys. Rev. C* **53**, 2661 (1996) [arXiv:nucl-th/9512036].
12. C. da Rocha, G. Miller and U. van Kolck, *Phys. Rev. C* **61**, 034613 (2000) [arXiv:nucl-th/9904031].
13. C. Hanhart, U. van Kolck, and G. Miller, *Phys. Rev. Lett.* **85**, 2905 (2000) [arXiv:nucl-th/0004033].
14. C. Hanhart and N. Kaiser, *Phys. Rev. C* **66**, 054005 (2002) [arXiv:nucl-th/0208050].
15. A. Gårdestig, talk presented at ECT* workshop *Charge Symmetry Breaking and Other Isospin Violations*, Trento, June 2005; A. Gårdestig, D. R. Phillips and C. Elster, *Phys. Rev. C* **73**, 024002 (2006) [arXiv:nucl-th/0511042].
16. D. A. Hutcheon *et al.*, *Nucl. Phys. A* **535**, 618 (1991).
17. P. Heimberg *et al.*, *Phys. Rev. Lett.* **77**, 1012 (1996).
18. M. Drochner *et al.*, *Nucl. Phys. A* **643**, 55 (1998).
19. P. Hauser *et al.*, *Phys. Rev. C* **58**, R1869 (1998).